Inverse Kinematics Guide

Some Help with Inverse Kinematics

What are Inverse Kinematics?

- Inverse Kinematics translates from an end effector position and orientation to joint angles
- $T_{0n} \rightarrow \theta_1, \theta_2, \ldots \theta_n$
  - $(x_{grip}, y_{grip}, z_{grip}, \theta_{yaw}, \theta_{pitch}, \theta_{roll}) \rightarrow \theta_1, \theta_2, \ldots \theta_n$
- We will be doing this analytically using geometry

Why do we care about Inverse Kinematics?

- Inverse Kinematics is much more useful than Forward Kinematics for what we wish to do
- We control the robot by joint angles, but we live and operate in a 3D – x-y-z world.
- Could you describe a straight-line in terms of $\theta_1, \theta_2, \theta_3, \theta_4$ ...? It is pretty hard to do.
- We need a way to translate our x-y-z world to the joint angles the robot needs
- Inverse Kinematics is the tool that allows us to do that.

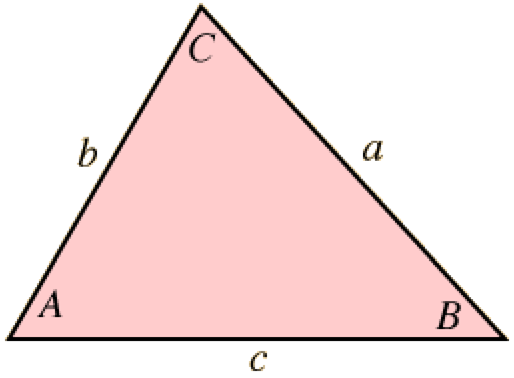
Background

- In class you have studied Numerical Inverse Kinematics which uses numerical methods to solve the problem
- In lab we will find Geometric Inverse Kinematics. We will use geometric relationships to find formulas to solve the problem
- With Forward Kinematics, there is a single solution to the problem, but with Inverse Kinematics, there are often multiple solutions
- We deal with multiple solutions by imposing constraints on our arms
- Sometimes there are multiple solutions to an angle, but not all solutions are good
  - Some solutions fail in certain configurations.

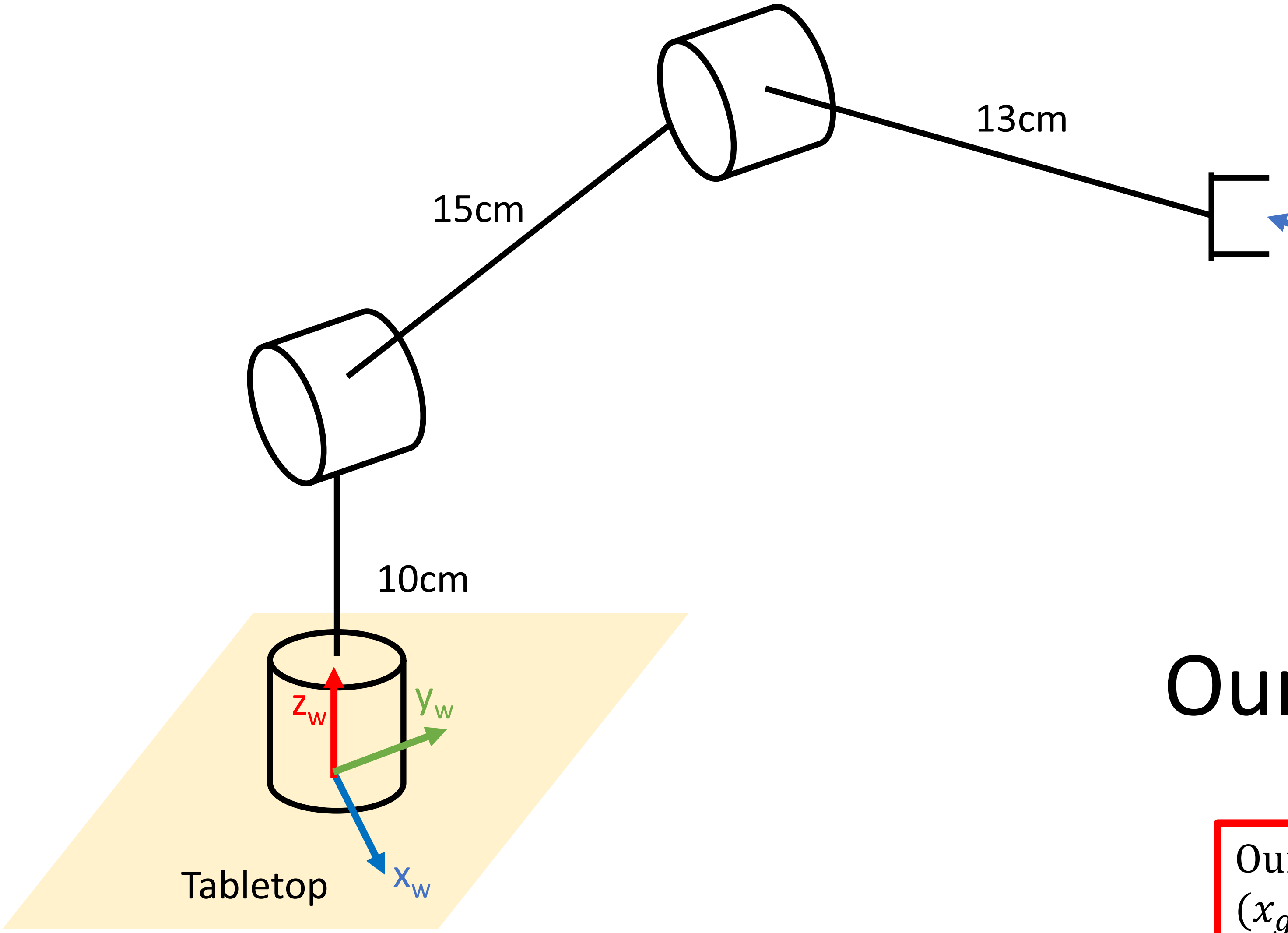
Useful Tools

- Basic Trigonometry – sines, cosines, and tangents
- Law of Cosines

$$c^2 = a^2 + b^2 - 2ab\cos C$$



- `atan2(y, x)`
    - Takes into account signs of x and y in arctan(y/x) and places angle in correct quadrant



Center of Gripper
(x, y, z)

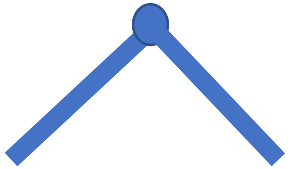
This is our given.

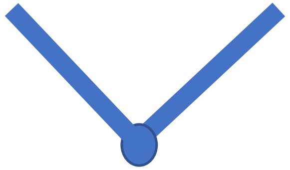
Our Example Robot

Our goal:
$(x_{grip}, y_{grip}, z_{grip}) \rightarrow \theta_1, \theta_2, \theta_3$

Set up

Elbow Down



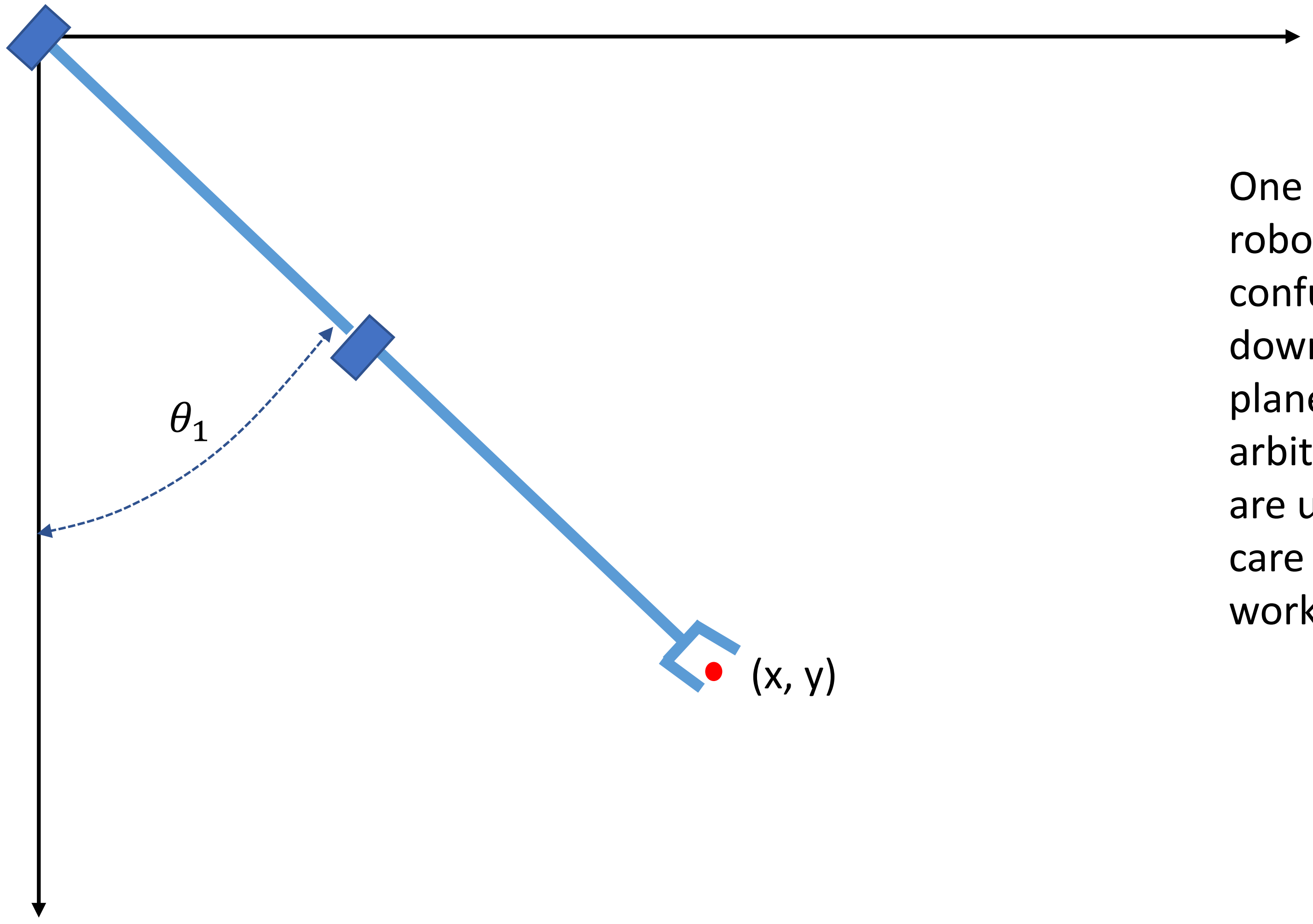


Elbow Up

- This is a simple robot, so we only need to worry about Elbow Up or Elbow Down configurations
  - Let's solve for Elbow Up
- Again, because we have a simple design, the solution order is not important
  - We will solve for θ_1, then θ_2, and finally θ_3.
  - It is important to know what information you have available, so you don't solve for θ_1 in terms of θ_2 and θ_2 in terms of θ_1.

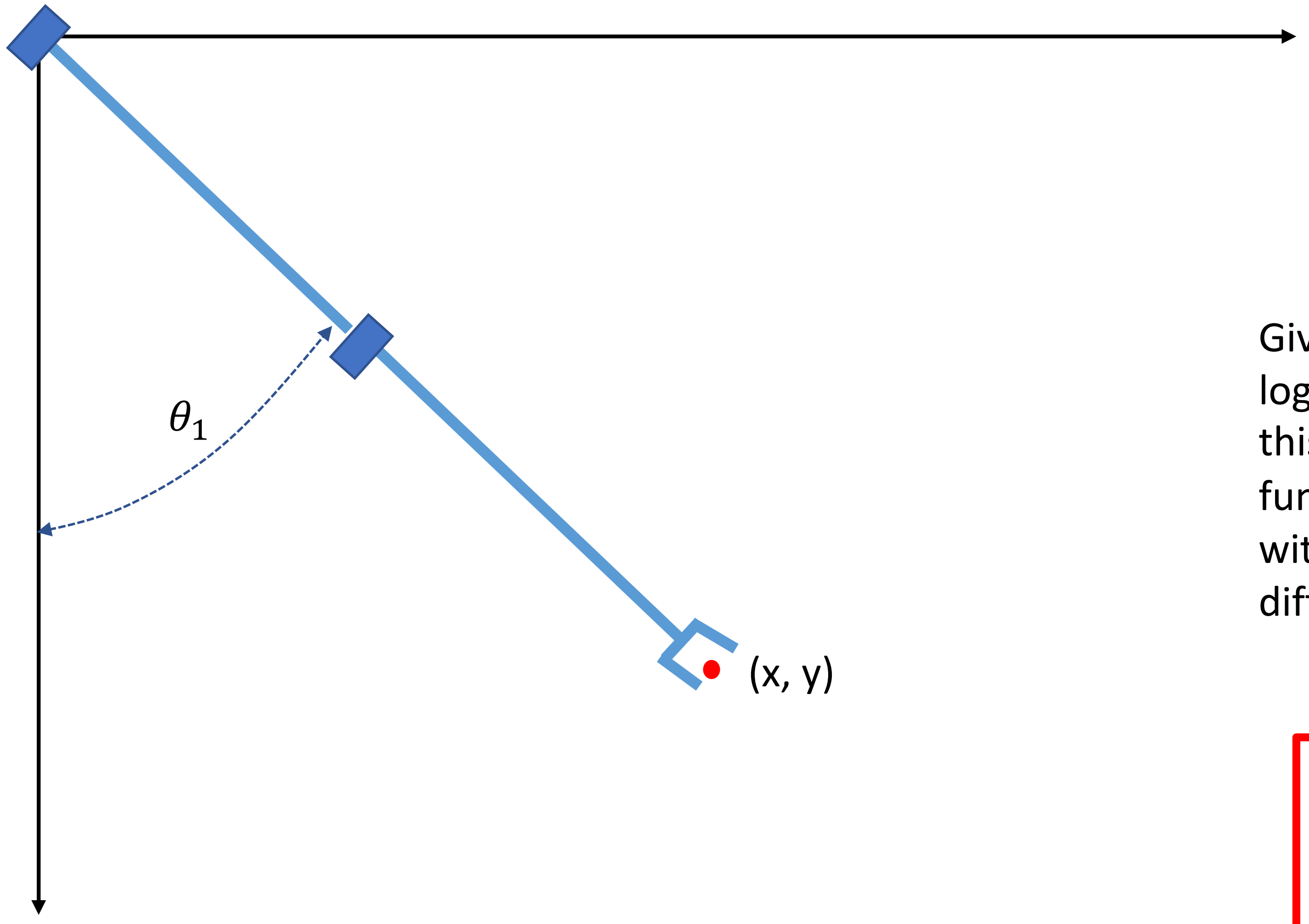
We want θ_1, what can we do?

Top View (x-y plane)



One useful technique is to project the robot on different planes to eliminate confusing details. Here we are looking down on the robot and at the x-y plane. We have placed the robot in an arbitrary configuration so our solutions are universal, but we still have take care that we can deal with the whole workspace.

Top View (x-y plane)



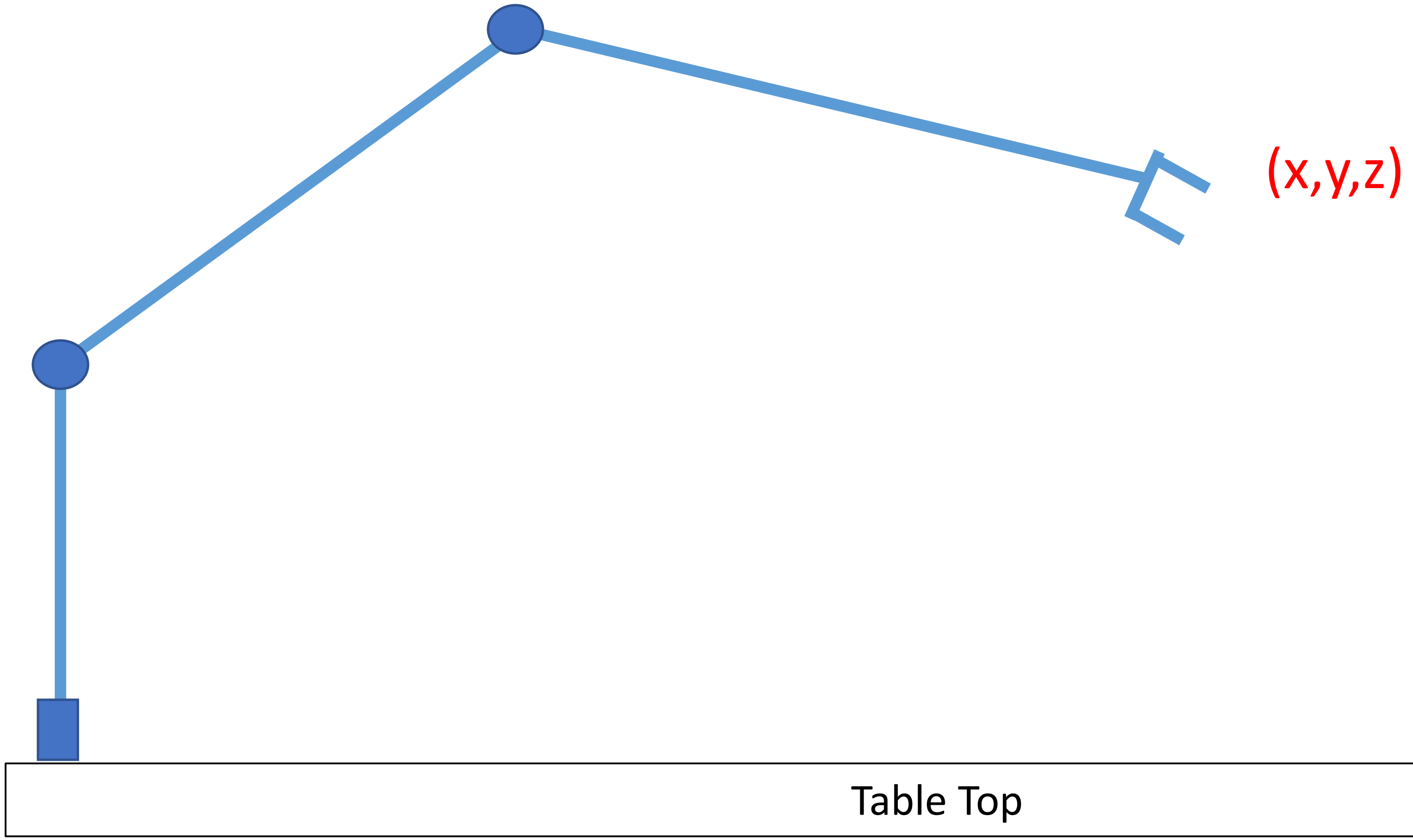
Given that we know x and y, it is logical to use an arctan to solve this problem. We use the `atan2` function in computing to deal with how tangent behaves in different quadrants.

Solution:

θ_1 = atan2(y, x)

“Side” View

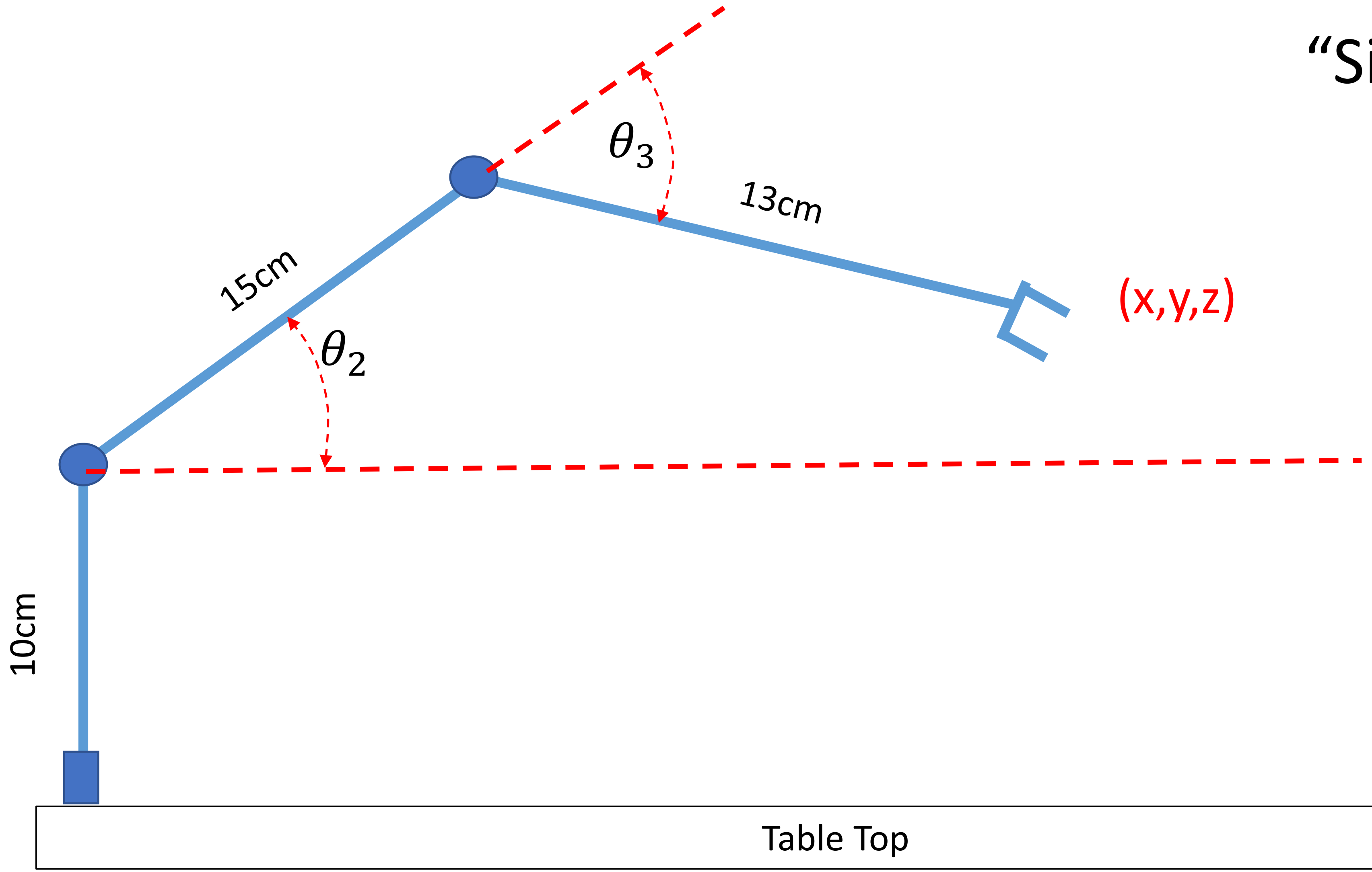
What plane is this?



We now look a different plane of the robot. This allows us to isolate θ_2 and θ_3. We want to start by putting in the known information.

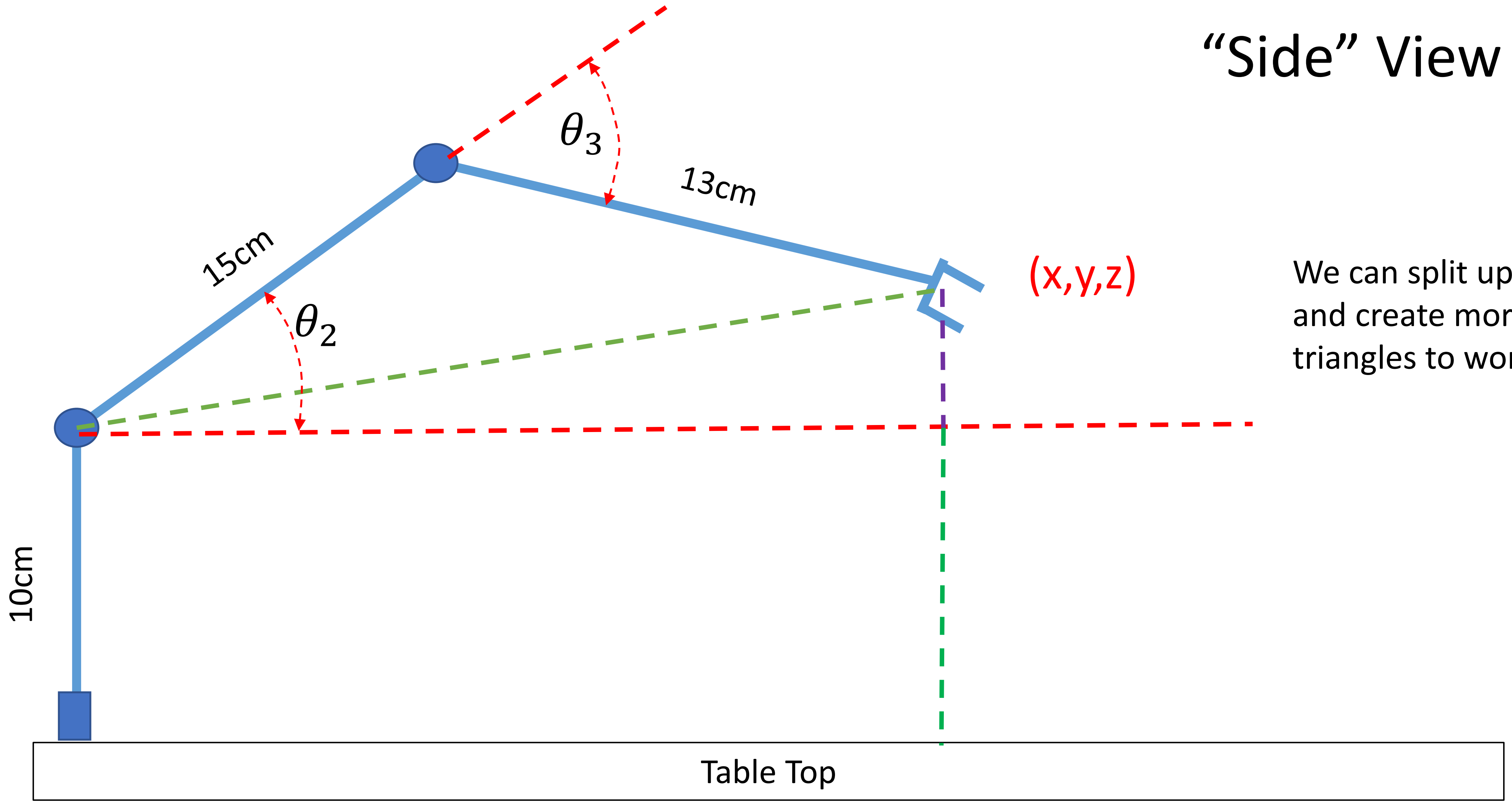
What lengths and angles do we know in this view?

“Side” View



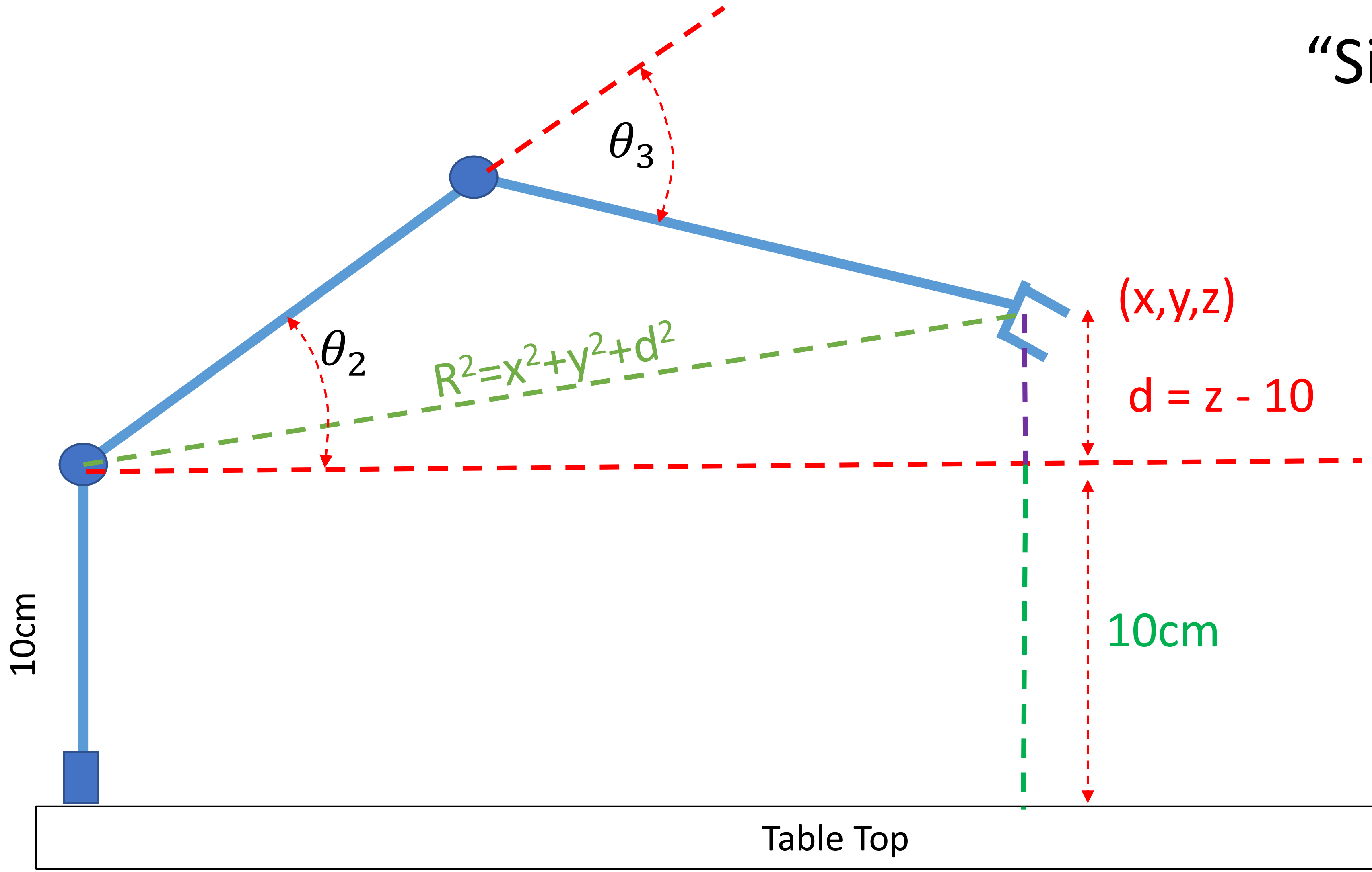
We can add in the angles - θ_2 and θ_3 and the lengths of the links.

“Side” View



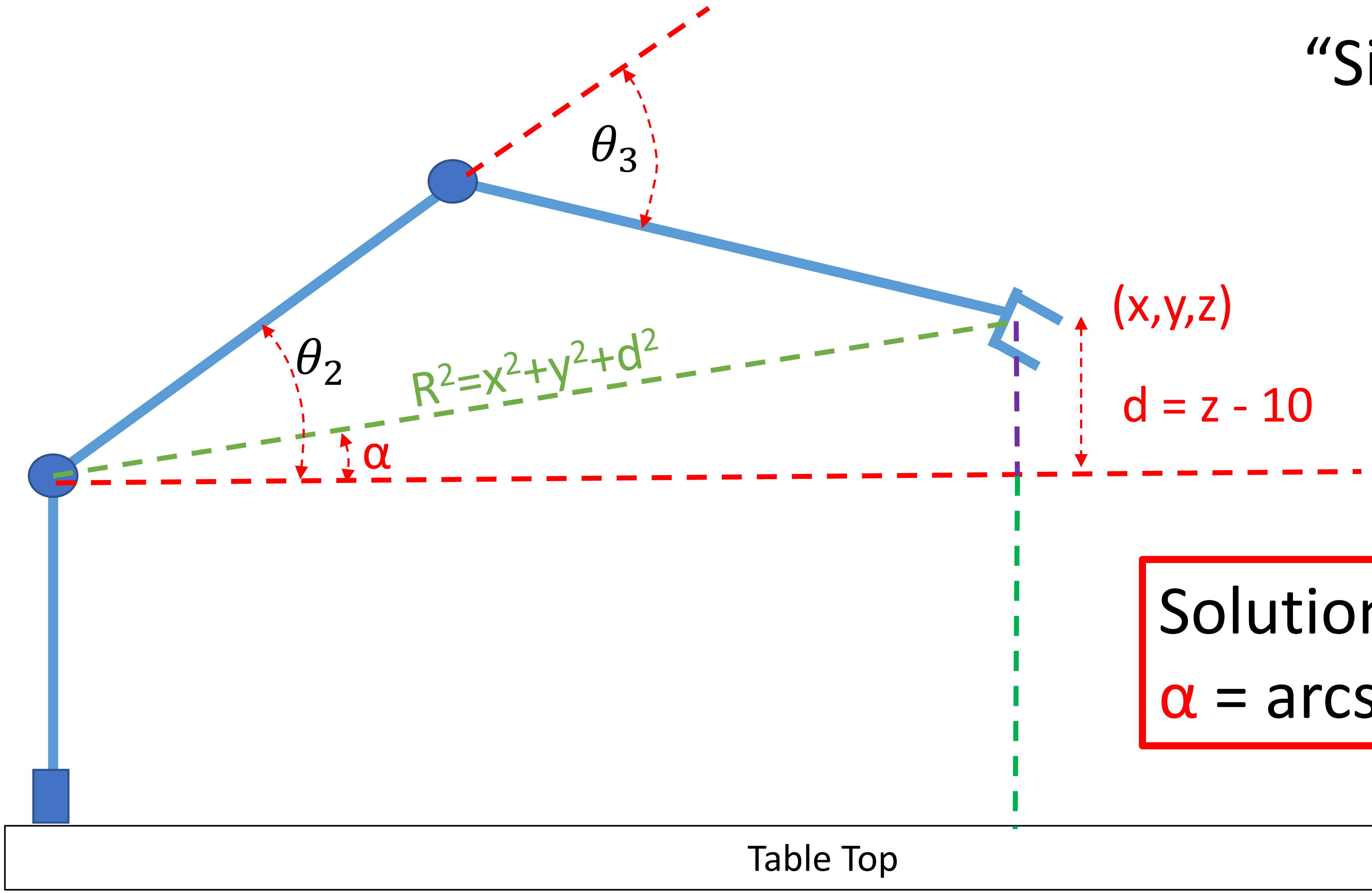
We can split up the area and create more triangles to work with.

"Side" View



Even with only (x, y, z) and the construction of the robot, we know a lot of information.

“Side” View

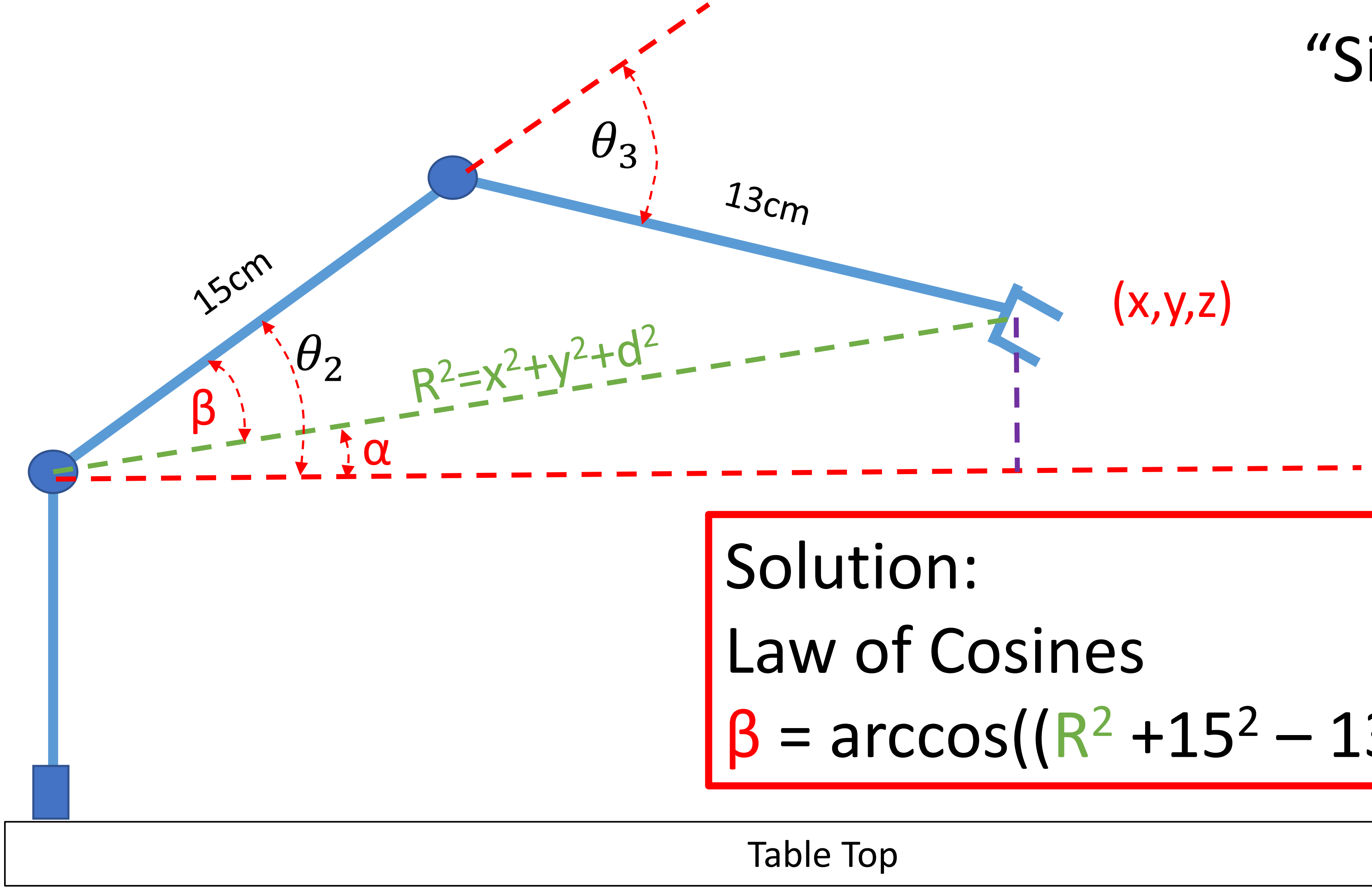


Let’s use this information to find α.

Solution:

$\alpha = \arcsin(d/R)$

“Side” View



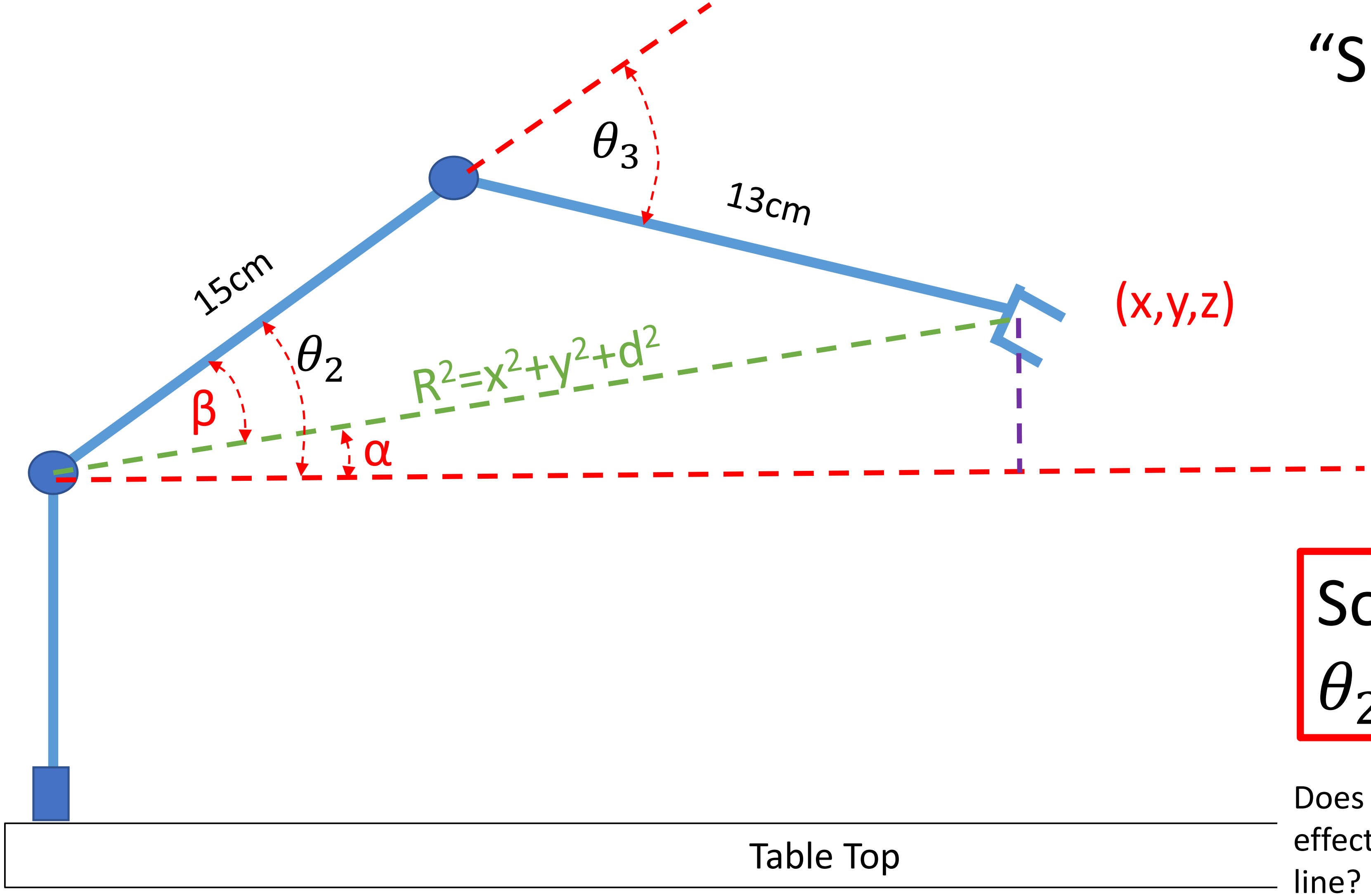
Now we know α, if we could find β we would know θ_2.

Solution:

Law of Cosines

$$\beta = \arccos((R^2 + 15^2 - 13^2)/(2*15*R))$$

“Side” View

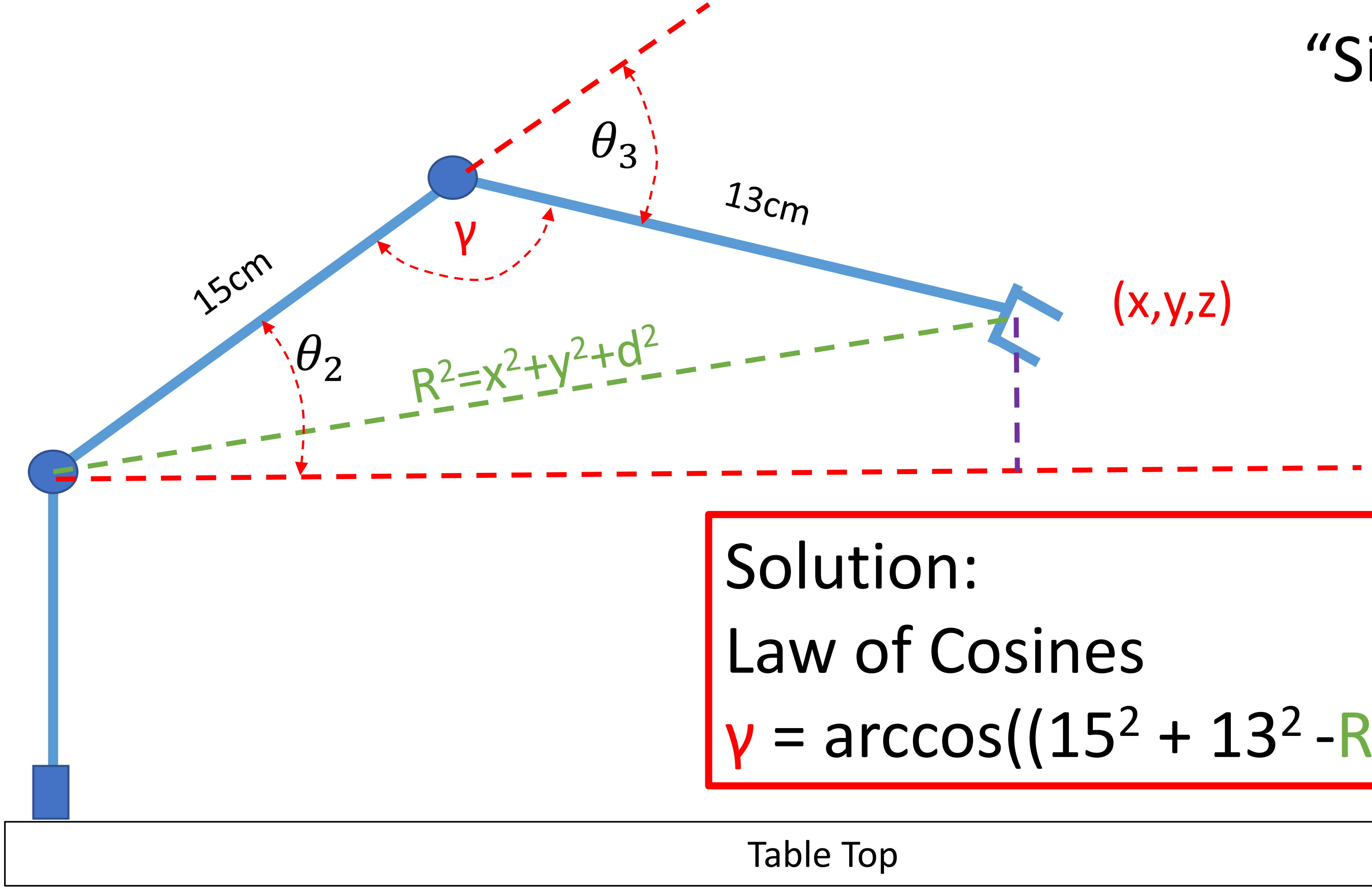


Solution:

$\theta_2 = \alpha + \beta$

Does this change if the end effector is below the red line? If θ_2 is negative?

“Side” View



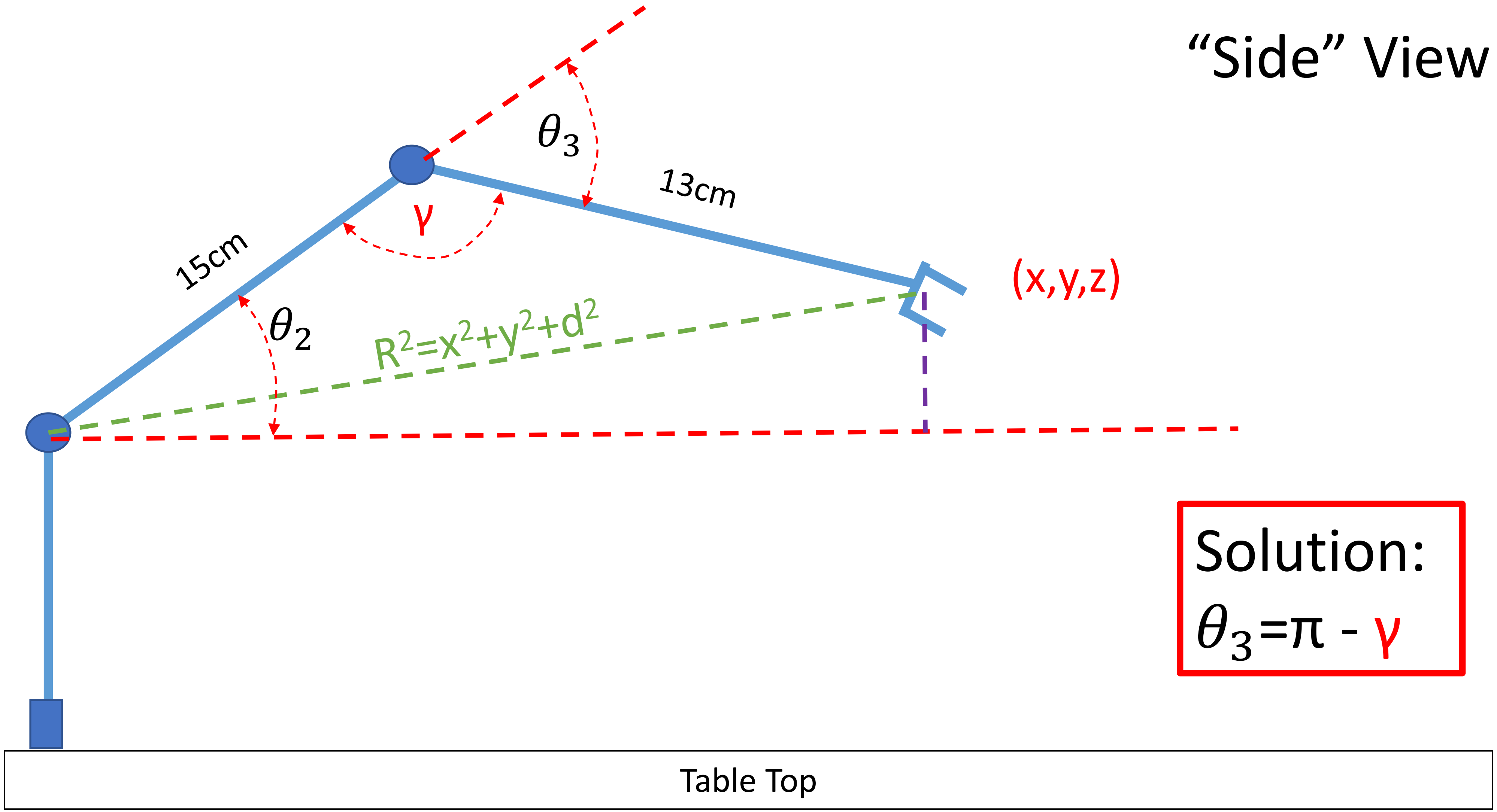
Now if we know γ, we could know θ_3.

Solution:

Law of Cosines

$\gamma = \arccos((15^2 + 13^2 - R^2)/(2*15*13))$

“Side” View



Solution:

$\theta_3=\pi - \gamma$

Summary of the Solution

Solution:

θ_1 = atan2(y, x)

d = z - 10

$R^2 = x^2+y^2+d^2$

α = arcsin(d/R)

β = arccos((R^2 +15^2 – 13^2)/(2*15*R))

θ_2 = α + β

γ = arccos((15^2 + 13^2 - R^2)/(2*15*13))

θ_3 = π - γ

We now have a solution that can calculate $\theta_1, \theta_2, \theta_3$ given $(x_{grip}, y_{grip}, z_{grip})$.

For Reference

“Side” View